An Introduction to Quantum Computing:
Quantum Computation of a Coin Flip

Johnathan Baird[α], Dr. Joshua Qualls[Ω]

[α] Craft Academy for Excellent in Science and Mathematics, [Ω] Department of Mathematics and Physics

SUMMARY

Quantum computing (QC) is a radically new approach to computation. Classical computing is based on the ideas of bits: strings of 1's and 0's on which we do arithmetic operations. QC, however, is based on quantum bits (*qubits*) and new physical operations. The situation is analogous to that of a candle and an LED flashlight—both are sources of light, but the principles by which they operate are fundamentally physically different. Until quantum computers are ubiquitous, we continue to make models and write algorithms for this powerful new technology.

Our project involves flipping a coin in secret. Depending on the outcome of the coin flip, you are given a qubit prepared in one of two different quantum states. You are allowed to measure the qubit once along any axis; depending on your measurement you have to guess whether the coin was "Heads" or "Tails".

Our project had several components:

(1) We learned the basics of QC and quantum information

(2) In the absence of a fully programmable quantum computer, we wrote a Python program to model/simulate this coin flipping exercise

(3) We derived the optimal success rate for both classical or quantum measurement. We then compared these theoretical results with both

(i) the results of performing the "quantum" measurements (modeled on our Python program), and

(ii) results of QC measurements using the IBM Q Experience

INTRODUCTION / BACKGROUND

Our problem begins with an Adversary ("ADV") secretly flipping a classical fair coin. Depending on the result of the flip, ADV will prepare one of two different quantum states:

If "Tails", ADV prepares the qubit in the state

$$|0\rangle \qquad \textbf{(Tails)} .$$

Physically, this means a spin (a kind of "angular momentum") measurement along the Z-axis will always give +1. This qubit is in a ("classical") state of definite spin.

If "Heads", ADV prepares the qubit in the state

$$\cos(x)\,|0\rangle + \sin(x)\,|1\rangle \qquad \textbf{(Heads)}$$

where "x" is a real number parameter that ADV will share with you. Physically, this state is a superposition: a spin measurement along the Z-axis will return a value of +1 with probability $\cos^2(x)$ and a value of -1 with probability $\sin^2(x)$.

When x = 0, we never expect to succeed more than 50% over repeated play.
When x = π / 2, the states are distinguishable; we succeed 100% of the time.

But what of arbitrary values of x? We assign measuring spin = +1 to guessing T and spin = -1 to guessing H. With this rule, we maximize the probability of success:

$$P_{success} = P(H)\cdot P(-1 \mid H) + P(T)\cdot P(+1 \mid T)$$

Measuring spin, how do we maximize chances of guessing the coin flip? Or,

Is there a "quantum" measurement that outperforms "classical" ?

For a classical model, the prepared qubit is actually and truly one of the classical states occurring with probabilities $\cos^2(x)$ and $\sin^2(x)$. Additionally, we are restricted to only measuring the qubit along the Z-axis. The best theoretical success rate for classical measurements is given by

$$P_{success,classical} = (3-\cos(2x)) / 4$$

Quantum mechanics, however, predicts a generically improved success rate. In order to take advantage of the power of quantum superposition, we choose an alternate axis for spin measurement that depends on the parameter *x*. The best theoretical success rate for quantum measurements is given by

$$P_{success,quantum} = (1+|\sin x|) / 2$$

METHODS AND RESULTS

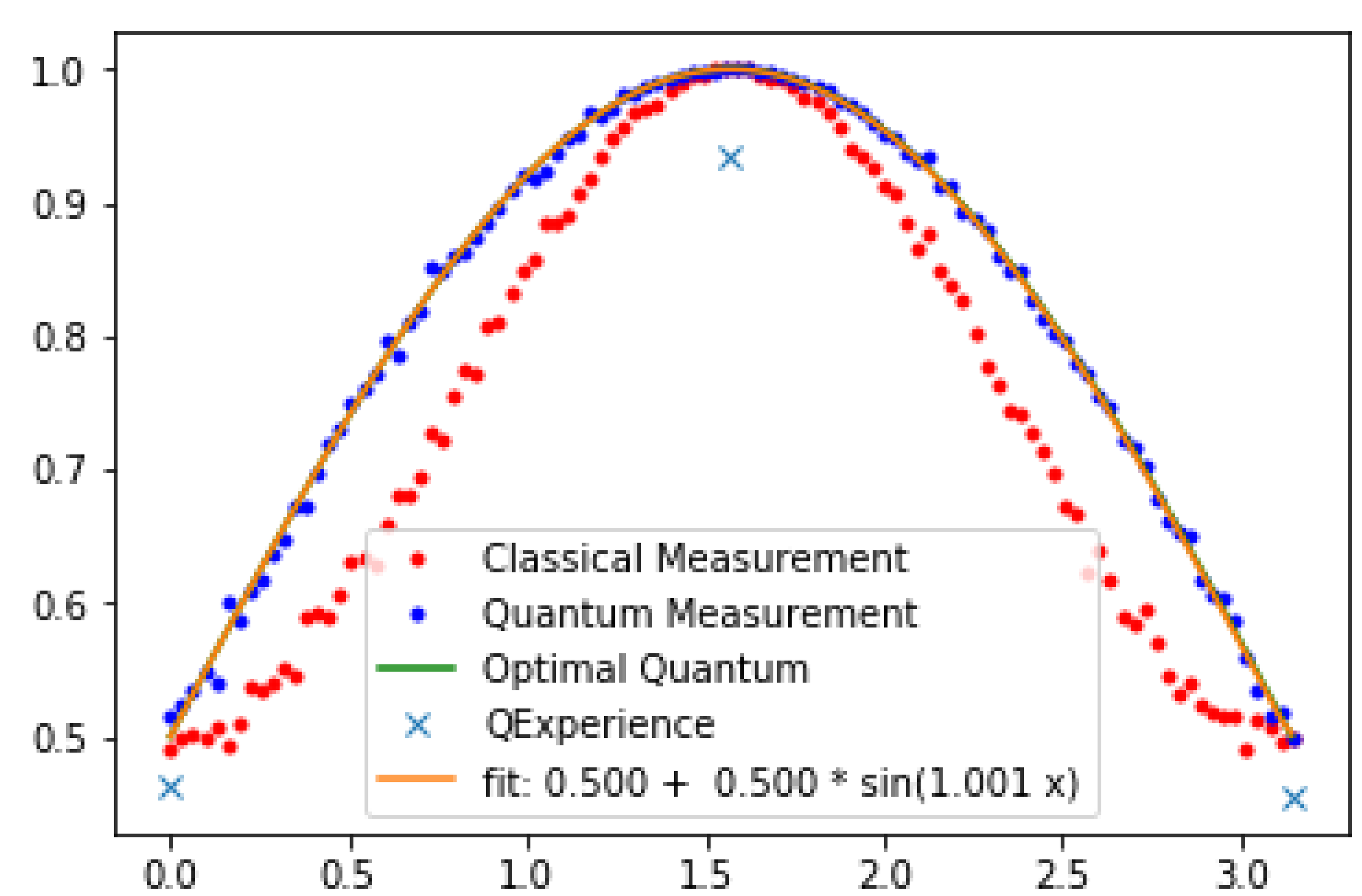

Figure 1

In Figure 1, the Simulated Quantum Measurement (in blue) is in almost perfect agreement in with the Optimal Quantum predictions (green). We fit a curve to this Simulated Quantum Measurement data (yellow), and this curve all but obscures the Optimal Quantum predictions. The curve fit parameters are in excellent agreement with theoretical values—the RMSE(Root Mean Squared Error) value for this experiment was approximately 0.0066 with the R-Squared value being 0.9982.

Our results also support our hypothesis that the probability of success using quantum measurements generically outperforms the probability of success using classical measurement. Our classical measurements (red) always give worse results than the Simulated Quantum Measurements.

Our results using the IBM Q Experience quantum computer gave inconclusive results. We were only able to measure three data points using this real quantum computer, and each of these data points gave a less than perfect correspondence.

CONCLUSION

This project was a success on almost all accounts. Our Simulated Quantum Measurements closely matched the theoretical predictions from quantum information theory. Additionally, the Simulated Quantum Measurements always outperformed the classical measurements (only along the Z-axis).

Our results from IBM Q Experience measurements were not, however, ideal. This could be due to many factors, but the most likely is that the IBM Q Experience quantum computer technology is still imperfect. It is extraordinarily challenging creating qubit states, maintaining them in isolation, having them interact/interfere, and performing measurements. With time, the accuracy of these measurements should improve. We would also like to take more data points, but we were limited by the quantum gates that IBM Q Experience lets you utilize. In order to measure different values of x, we need greater control over the quantum circuit. This means that we will either have to devote significantly more time to writing out quantum circuits appropriate for IBM Q Experience, or consider another quantum computing problem more appropriate for their resources.

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Libraries used in python code: Random, Math, NumPy, PyPlot, SciPy.